Stockton University
Mathematical Mayhem 2016
Group Round

April 9, 2016

Name: ______________________________

Name: ______________________________

Name: ______________________________

High School: ______________________________

Instructions:

- This round consists of **5** problems worth **16** points each for a total of **80** points.
- Each of the 5 problems is free response.
- Write your complete solution in the space provided including all supporting work.
- No calculators are permitted.
- This round is **75 minutes** long. **Good Luck!**

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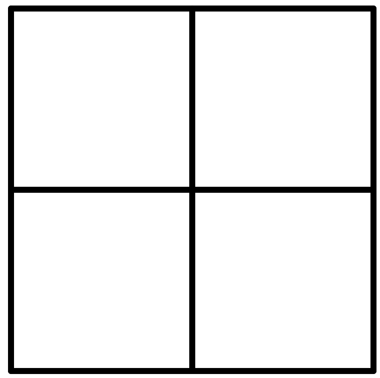
Problem #	1	2	3	4	5	Total
Points Earned						

♠ Group Round ♠

Problem 1. How many rectangles are there in a 4×6 grid of squares? How many in an $m \times n$ grid? For example, there are 9 rectangles in the 2×2 grid shown below.



Problem 2. The coordinates of A, B, and C are $(5,5)$, $(2,1)$ and $(0,k)$ respectively. Find the value of k that makes the sum of the lengths of line segments AC and BC a minimum.

Problem 3. Tim has a solid wooden cube whose side lengths are integers greater than 2. He paints the entire surface of the cube red. Then, with slices parallel to the faces of the cube, Tim cuts the cube into $1 \times 1 \times 1$ cubes. A certain number of the small cubes are completely free of paint (x). A certain number of the small cubes are painted red on only one side (y). A certain number of the small cubes are painted red on two sides (z). There are also 8 cubes that are painted red on 3 sides, but these are not invovled in our problem.

(A) If y is twice as big as x, what were the dimensions of Tim's original cube?

(B) If x is twice as big as y, what were the dimensions of Tim's original cube?

(C) If $y + z$ is 33% of x, what were the dimensions of Tim's original cube?

Problem 4. How many pairs (x, y) of non-negative integers with $0 \leq x < y$ satisfy the equation

$$5x^2 - 4xy + 2x + y^2 = 624?$$

List all of the pairs.

Problem 5. Consider a currency that has 100 cent (dollars), 50 cent (half-dollars), 25 cent (quarters), 10 cent (dimes), 5 cent (nickels), and 1 cent (pennies) coins. To specify a collection of coins, we will use an ordered list of six numbers (D, H, Q, I, N, P) where

D	is the number of dollars,
H	is the number of half-dollars,
Q	is the number of quarters,
I	is the number of dimes,
N	is the number of nickels, and
P	is the number of pennies.

For example, $(0, 1, 2, 0, 0, 0)$ and $(0, 0, 0, 0, 0, 100)$ are two different collections of coins that make 100 cents, the first by a one half-dollar coin and two quarters and the second by one hundred pennies. That is, you can make 1 dollar with exactly 3 coins and you can also make 1 dollar with exactly 100 coins.

Is it possible to make one dollar using each number of coins between 1 coin and 100 coins? If so, explain why. If not, list the numbers of coins that fail.