Teddy Mishura

Self-Avoiding Walks on a Rectangular Grid: Final Report

Over the course of my Fellowship project, many of the proposed objectives were obtained – a formula for the number of self-avoiding walks (SAWs) on rectangular grids of fixed width but variable height was achieved (specifically for grids of width 0, 1, 2, and 3) and certain "families" of SAWs – i.e. types of SAWs that all share a common property – were studied, while formulas were found for their behavior on grids of both variable height and width. Dr. Forrest and I first spent our time meeting together in an attempt to analyze and study the behavior of SAWs on grids of small width so that the knowledge gained from that pursuit could be put to use attacking the problem of SAWs contained on grids of large and even variable width.

During the course of our research, we developed a recursive technique to completely enumerate SAWs contained on grids of width equal to or lesser than 3; with this knowledge in hand, we extended our technique to grids of larger widths by utilizing it to enumerate all SAWs of a certain family on grids of larger width. Due to this family of SAWs not comprising all possible SAWs on grids of width larger than 3, our approach to extension does not completely enumerate SAWs on grids of such width; however, it does provide a remarkably accurate estimate in regards to the total number of SAWs contained within such mentioned grids. As such, further effort was spent to refine the technique to attempt to lower the margin of error as much as possible. In certain cases we have been able to lower the error margin of our enumerative technique to as little as 1%.

Time was also spent analyzing the behavior of a family of SAWs that we called "backtracking" SAWs – named as such because these SAWs are only allowed to move in the down direction (which we call "backtracking") of the grid they are contained on a fixed number

of times. The enumerative technique we developed for SAWs on grids of small width was also extended to study this family of SAWs, but with much greater success. We were able to extend the technique to completely enumerate all backtracking SAWs contained on a grid of both variable height and width. However, the recursive nature of the technique necessitates that immense computational time be utilized in order to render solutions for even marginally large grids (such as 7x7, for example). Therefore more time was spent in order to refine the technique and cut down on computational time; however, as this was the most recent advance of our research, the progress made so far has been little, and Dr. Forrest and I plan to continue working on this area of the project come the following semester.

Further plans for the project after the funding period are well underway: Dr. Forrest and I plan to create an independent study class where I can focus on continuing this research during the semester, where my main goals will be to both further refine the technique Dr. Forrest and I have created and to study in-depth certain families of SAWs with restricted behavior to see if they can be used to model the behavior of SAWs as a whole. I will also devote time to further perfect the technique we have developed to enumerate backtracking SAWs. I am confident that within the four months of the following semester that I will make significant progress on refining both of the techniques that Dr. Forrest and I have created, and that I will make headway into understanding the families of SAWs I intend to research. Dr. Forrest and I have already begun to meet in regards to discussing such families of SAWs and will continue to do so during the semester due to the independent study class we will create. As such, continuation of the project is assured and I am confident that the time I will spend researching this problem will be fruitful in its results.

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Dissemination venues: The results of the project were presented at both Stockton University, located in Galloway, New Jersey, and at Moravian College, located in Bethlehem, Pennsylvania. At both times the project was presented to an audience of undergraduate math majors and math faculty. Near the end of the funding period, the project was presented at the Mathematical Association of America's MathFest, a large professional gathering of mathematicians designed to foster undergraduate mathematical research. Presentations ranged from lengths of 15 to 90 minutes each.

Expenditure Report: $0.